A Most Merry and Illustrated Explanation of Einstein's Special Theory of Relativity

And a Simple Resolution of the Twin Paradox Regardless of Reference Frame (And No Space Time Diagrams Needed)

Everybody has heard of Albert Einstein and that he discovered the Theory of Relativity. Some have even heard that Relativity leads to the famous ***Twin Paradox***. But they also note that although the Twin Paradox is often described, it's rarely explained in a nice simple manner.

After divorcing Mileva, Albert had some difficulties in the single bars around Zurich.

Now if you'd first like some extra background in what Albert did, and why he did it, there is a rather lengthy explanation at http://www.coopertoons.com/education/specialrelativity/relativity_twinparadox.html.

But if you feel you ***are*** versed enough in the basics of Albert's theory, to tackle a simple explanation - and ***resolution*** - of the Twin Paradox, read on!

The Fundamental *Measured* Speed of Light

What made Albert famous was that he had proposed an explanation why the speed of light ***as measured*** was always the same regardless of how fast the individual was traveling.

But why, you ask, should it change?

Well, if you were moving at a certain speed through space - we'll call your speed ***v*** - and you shone a flashlight in the direction you were traveling, then you would think the total speed of the light beam would be your speed, ***v***, plus the usual speed of light (which we always write as ***c***).

Expected Speed of Light = v + c

But that's not what you see. The speed of a beam of light ***as measured by anyone*** is always just ***c***. Many physicists of the late 19th century came up with various explanations why this is, but the explanations sounded too much like 19th century physics.

But Albert's explanation - given five years into the 20th century - was we needed no explanation. There ***is no absolute speed of light***, he said, that is a ***fundamental constant*** of nature. What ***is*** a fundamental constant is the speed of light ***as measured*** regardless of the speed of the observer or the speed of the source of the light. Or to put it in fancy pants language, the speed of light is ***invariant with reference frame***.

That is the ***Principle of Relativity*** in a nutshell.

Even today there are some people who object to relativity on moral grounds. Some even claim relativity is no longer accepted by scientists. Now in addressing such assertions we will not stray from parliamentary politeness, but the claim is complete poppycock, horse hockey, and bull-feathers. Relativity is regarded as one of the most important discoveries in the history of science.

As far as people objecting to Relativity on moral grounds, we will say that the briefest perusal of their - we'll call them "essays" - makes it obvious what the problem is. They are confusing ***Relativity*** with ***Relativism***.

Relativism is indeed a philosophy that claims there is no absolute morality and - and here we quote a famous modern philosopher - that "our true knowledge is made true not by objects independent of us, but by our own ideas, perspectives, theories, and cultures." Rest assured, ***mes amis***, Relativity and Relativism are not related in the slightest.

In fact, Relativity is nothing more than a simple ***mathematical consequence*** of the now ***well verified experiment*** that the speed of light does not change with the speed of the reference frame. The experimental result has been confirmed repeatedly. The rest is simple middle school mathematics.

Yes, **simple** ***middle school mathematics***.

Special Relativity

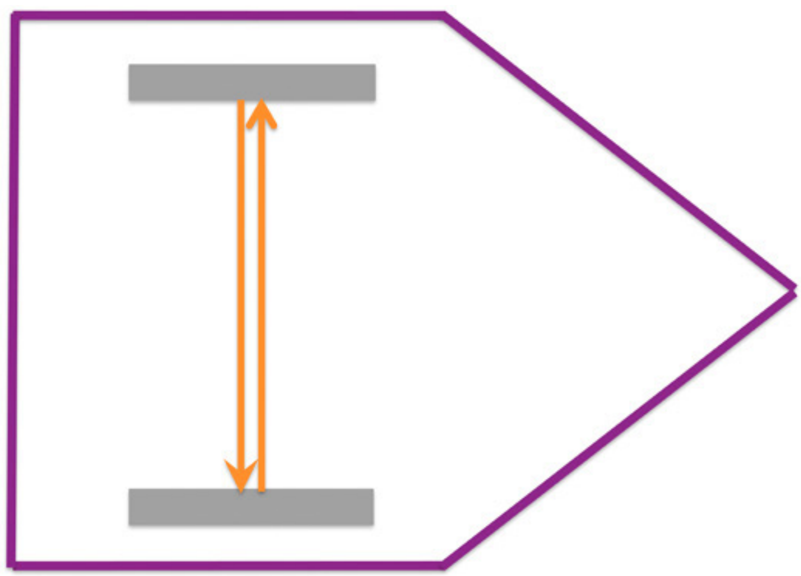
This essay, we should point out, is about the ***Special*** Theory of Relativity. That is we are talking about objects moving at constant speed and in a straight line. Later Albert developed the ***General*** Theory of Relativity dealing with motion not in a straight line or with changing speeds. But the math isn't as simple so we'll skip the General Theory for now.

First, don't get us wrong. We are not trying to be condescending to people who have trouble with Relativity. We don't say Relativity isn't strange.

For instance, relativity tells us ***as you speed up, time slows down***. But it doesn't seem to ***you*** that ***your*** time slows down. But to someone who isn't going as fast as you are, ***your*** time slows down.

Let us elaborate.

Now Joe Blow is on a Spaceship, and he keeps time with a ***light clock***. A light clock is simply a beam of light bouncing between two mirrors. We'll designate the time it takes for the beam to go from one mirror to the other and back as a "Spaceship second" although it's obviously much faster than that (or it's a mighty big Spaceship).

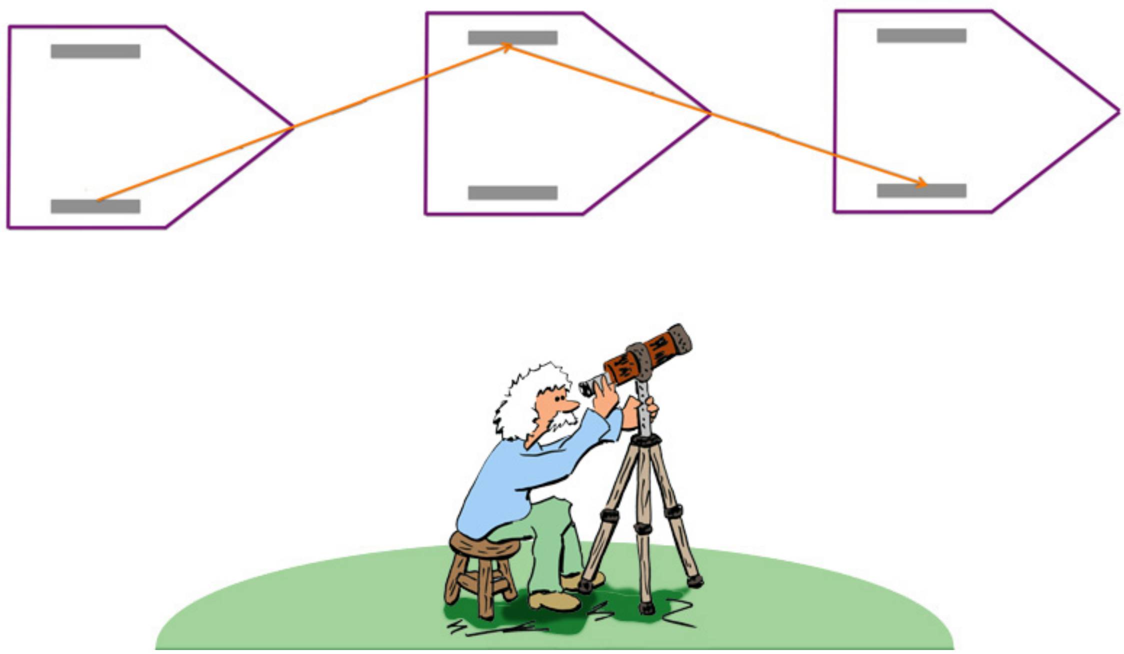


The Spaceship Light Clock

But suppose the Spaceship is now flying past the Earth at some superfast speed. And someone on Earth looks up with a telescope and follows the beam of light.

What does the Earth-bound observer see? Well, he will see the light beam travel ***further*** than Joe Blow on the Spaceship did.

That's because the light beam doesn't go straight up and down when seen from Earth. Because the Spaceship is moving, the light follows a zig-zag path.



Someone on Earth looks up.

Now consider this. If the light ***as measured on the Spaceship*** travels a shorter distance than ***as seen by the observer on Earth***, but both see the light beam traveling at the same speed, then ***time must be slower on the spaceship as measured on earth***.

We easily can prove this with our middle school math. In doing so we derive an equation for what is called ***time dilation***. It's very simple, really.

What we do is look at the ***distance*** traveled by the light from both the Spaceship reference frame and the Earth reference frame. It's also simpler if we just look at the time it takes the light to travel from one mirror to the other rather than bouncing back and forth.

Now remember your (middle school) physics. The distance you travel is simply your speed (velocity) multiplied by the time you travel.

$$\textbf{distance} = \textbf{velocity} \times \textbf{time}$$

At first we'll consider the time ***that the light takes to go from one mirror to the other. As measured on the Earth***, we'll call that time $\mathbf{t_1}$. We calculate the distance the Spaceship travels during that time, $\mathbf{d_1}$, as:

$$\mathbf{d_1 = v \times t_1}$$

where of course **v** is the velocity of the Spaceship as seen on Earth.

Now the distance the light travels ***as measured on the Spaceship*** is the speed of light, c, times the time it takes the light to travel between the mirrors - ***but only the vertical distance***. Call that distance, d_2, and the time as measured on the Spaceship we'll call t_2.

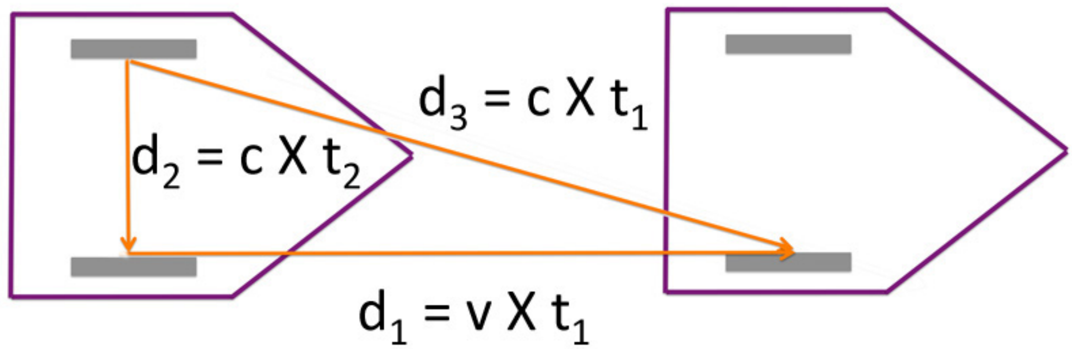
So that means on the Spaceship the light travels:

$$\mathbf{d_2 = c \times t_2}$$

But how far does the ***Earth observer*** see the light travel? Well, it's the speed of light, c, multiplied by the time it takes for the ***observer on Earth*** to see the light pass between the two mirrors. The time, of course, is the time measured on Earth, t_1. We'll call the new distance d_3.

$$\mathbf{d_3 = c \times t_1}$$

And all of these distances, speeds, and times, form a right triangle:



Space Clock: From One Reference Frame to Another

And are related by the Pythagorean Theorem:

$$\mathbf{d_1^2 + d_2^2 = d_3^2}$$

which is the same as:

$$\mathbf{(vt_1)^2 + (ct_2)^2 = (ct_1)^2}$$

And now ***mit ein bischen Mittelschule Algebra:***

$$\mathbf{(vt_1)^2 + (ct_2)^2 = (ct_1)^2}$$
$$\mathbf{v^2t_1^2 + c^2t_2^2 = c^2t_1^2}$$
$$\mathbf{c^2t_2^2 = c^2t_1^2 - v^2t_1^2}$$
$$\mathbf{c^2t_2^2 = t_1^2(c^2 - v^2)}$$
$$\mathbf{t_2^2 = {}^{t_1^2(c^2 - v^2)} / {}_{c^2}}$$
$$\mathbf{t_2^2 = t_1^2(1 - {}^{v^2}/{}_{c^2})}$$

And we get the final formula:

$$\mathbf{t_2 = t_1\sqrt{1 - (^v / _c)^2)}}$$

where t_1 is the time in the stationary reference frame, and the time t_2 is for a moving reference frame traveling at velocity v.

Now notice that if the speed of the reference frame, v, is greater than the speed of light, c, then the number under the square root sign, $1 - (^v / _c)^2$, would be a ***negative number***. But a negative square root is not permitted. This brings us to one of Albert's conclusions that you probably know.

Nothing can move faster than the speed of light.

And since v is always smaller than c, then the number under the square root sign, $1 - (^v / _c)^2$, is ***less than 1***. Or simply written:

$$\mathbf{t_2 < t_1}$$

And that proves what we said.

Time slows down for a moving reference frame.

This then leads to another conclusion:

If time slows down for someone in a moving reference frame, they travel a shorter distance when moving between two points.

Therefore

Length contracts as you move faster.

Given the relationship between distance, speed, and time, the length contraction equation is:

$$\mathbf{l_2 = l_1\sqrt{1 - ({}^{v} / {}_{c})^2)}}$$

where $\boldsymbol{l_1}$ is the length in the stationary reference frame and is the length $\boldsymbol{l_2}$ of the moving reference frame.

We again emphasize. These conclusions are an inescapable consequence from a well verified experimental result and middle school math.

Not the Albert Factor

Now the equation for time dilation gives us what is called - not the Albert factor - but the ***Lorentz factor***, named after the physicist who came up with the formula ***before Albert did.***

That is,

$$\mathbf{t_2 = t_1\sqrt{1 - ({}^{v} / {}_{c})^2)}}$$

which can be written as:

$$t_2 = t_1\boldsymbol{H}$$

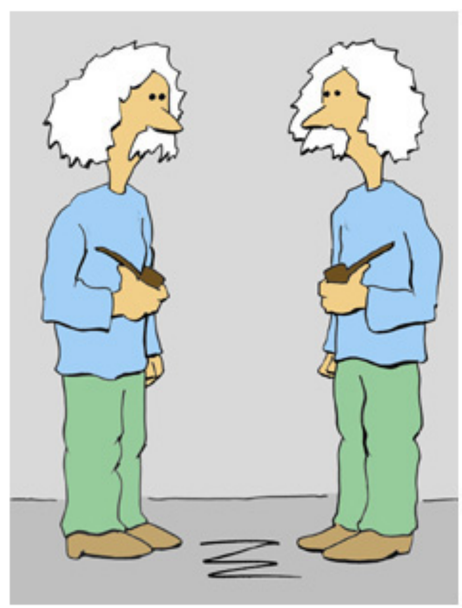
where

$$\boldsymbol{H} = \sqrt{\mathbf{1 - ({}^{v} / {}_{c})^{2})}}$$

(We use ***H*** since Lorentz's first name was Hendrik).

Although today you'll read how some people rant that Albert "plagiarized" his equations from Hendrik, the truth is that all physicists of the time knew the equations were from Hendrik's earlier papers. Today's rigorous standards of citation were neither expected nor needed. For his part, Hendrik praised Albert's theory, and Albert said he thought Hendrik was one of the greatest physicists of all time. So if it doesn't bother them, it shouldn't bother us.

The Twin Paradox



Now we come to what causes many people trouble - and believe it not, we also mean it bothered some famous scientists. We are referring to one of the most famous "thought experiments" in science.

That's the ***Twin Paradox***.

Suppose you have two twins. One gets in a Spaceship and the other stays on Earth. Let's say the Earth Twin sees the Spaceship head off at 185,000 miles per second - that is nearly the speed of light. By Earth's time, the Spaceship travels 25 years (measured on Earth), turns around, and comes back. So he spent 50 years in space.

But in the moving Spaceship, the time is calculated with Special Relativity. So we get:

$$\textbf{Trip Time (on Spaceship)} = 50 \textbf{ years} \times \sqrt{1 - ({}^{185,000}/_{186,000})^2)}$$

$$= 50 \textbf{ years} \times \sqrt{1 - 0.989276217}$$

$$=50 \textbf{ years} \times \sqrt{0.010723783}$$

$$=50 \textbf{ years} \times 0.1035557$$

$$= 5.2 \textbf{ years.}$$

So if the Twins were 25 years old at the start of the trip, the Earth-Twin is now 75 years old. But his Twin - who flew in the rocket - is only 30 years old!

But here is the problem:

If there are no absolute reference frames, the Space Twin could consider *himself* to be stationary. In that case it is the *Earth* that is moving off at 185,000 miles per second. So the *Earth* returns after a total of 50 years. So it is the *Earth Twin* who has aged five years!

Now we said time is not an absolute in Relativity. But that does ***not*** mean that a person can be 30 years old and 75 years old at the same time. So we seem to have found a ***true contradiction***. And contradictions, the mathematicians tell us, refute the premise.

Well, when some people read this, they conclude relativity is hokum, poppycock, and hogwash.

But hold on there, pilgrim. Not so fast.

Remember that Relativity is a ***simple mathematical consequence*** from an ***experimental observation***. Not only an experimental observation, but an experimental observation that has been repeatedly verified by independent methods.

So how do you resolve the Twin Paradox? Well, there's a number of ways you'll read on the Fount of All Knowledge (the Internet) - and sometimes even in those non-electronic devices with white flappy things in the middle (can't remember what they were called).

The first way out is to say that the Twin Paradox is not a valid thought experiment. Remember, Special Relativity is limited to motion traveling in a straight line and at a constant speed. So you can't use special Relativity when talking about a round trip.

Another way to explain the paradox is to say that it is the person who moves relative to the ***inertial reference frame*** and who ***undergoes acceleration*** that gets younger. So the Spaceship leaves, heads off, turns around, and comes back, leaving the reference frame and undergoing acceleration. The Space Twin is the younger. Note this - quote - "explanation" - unquote - doesn't really explain anything. It just restates the observation.

There is also a popular approach where you just smirk at the question and say, well, you need to learn about Albert's ***General*** Theory of Relativity. So just learn tensor analysis, take advanced physics courses, and you'll understand.

The truth, though, is if you keep track of everything and just use the Lorentz factor and middle school math - everything works out.

Reference Frame 1: The Frame Where the Earth Stood Still

First we must emphasize, actual physical reality must be consistent. The Twins can't both get younger and older. Reality is reality.

So we have to ask: Is it possible for the two twins to ***agree*** that the Earth Twin is older and the Space Twin is younger ***regardless of who we consider moving?*** The answer is yes.

First we start out with the Earth standing still. And to keep track of the distance, we will have the Spaceship travel to a ***Planet***.

What we're going to do next is to let the two Twins look at each others' clock. Of course, the clocks have to be viewed on a television set, and the images are sent via radio waves. The radio waves travel, of course, at the speed of light.

We also need to keep in mind the three points of Special Relativity.

> **First, *time slows down* for an object moving relative to another when measured from the "stationary" reference frame.**
>
> **Second, *length contracts* for an object moving relative to another, again when measured from the stationary reference frame.**

Third, adjustments are made using the Lorentz factor, $\sqrt{1 - ({}^{v}/{}_{c})^2}$

To keep the calculations simple, we'll keep with the numbers used by a well-known physicist when talking about Relativity. That's to let the distance the Space Twin travels be 6 light-years and also to let the Spaceship travel at 111,699 miles per second. That's 60 % the speed of light. This is 0.6 light-years per year which astronomers and physicists write as ***0.6c***. Light, of course, travels at one light year-per year or ***1c***.

Now, as the psychiatrist said, we can begin, yes?

Gentlemen, Start Your Engines!

So what we have now is the Space Twin taking off in the Spaceship and head toward the Planet.

Similarly the Earth Twin remains on Earth but can watch the Spaceship and monitor the Spaceship's clock on his television.

So the Earth Twin watches the Space Twin set out for the Planet which is 6 light-years from Earth. And the velocity is 0.6c.

So it takes the Space Twin:

$$^{\text{6 light-years}} / _{\text{0.6 light-years per year}} = \textbf{10 years}$$

to make the outbound trip.

And does the Space Twin also make his trip in 10 light-years?

Nope. Remember according to the Earth Twin, the Space Twin's Clock slows down by the Lorentz factor:

Time for Space Twin = Time for Earth Twin × $\sqrt{1 - (^{v}/_{c})^2}$

So plug in the values and we get:

Time for Space Twin = 10 years × $\sqrt{1 - (^{v}/_{c})^2}$

which is

Time for Space Twin = 10 years × $\sqrt{1 - (^{0.6c}/_{c})^2}$

and is:

Time for Space Twin = 10 × $\sqrt{1 - 0.6^2}$

= 10 × $\sqrt{1 - 0.36}$

= 10 × 0.8

= 8 years

So here the Lorentz factor is 0.8, and the Space Twin's clock will read 8 years when it reaches the planet, not 10. But at that time, the Earth clock will still read 10 years.

As both the Space Twin and the Earth Twin are watching the Space Clock, they both agree that the space clock reads 8 years when the Spaceship reached the planet.

[Note: The Earth Twin does not ***see*** the Space Twin reach the planet after 10 years because there is a lag of the light and radio waves. It is an interesting exercise to figure out what and when the Twins see, but we'll pass that for now.]

And on the trip back?

Again the Earth Twin sees the Space Twin's clock go from 8 years to 16 years while his own clock goes from 10 years to 20 years. Similarly the Space Twin watches the Earth clock go from 10 to 20 years while his goes form 8 to 16 years.

So to summarize everything so far:

The Earth Twin sees his clock run from 0 to 20 years by the time the Spaceship returns. And he agrees the Space Twin's clock ran from 0 to 16 years.

The Space Twin sees his clock run from 0 to 16 years during the duration of his trip. And he agrees the Space Twin's clock ran from 0 to 10 years.

Everyone agrees the Space Twin is four years younger when he gets back to Earth.

And now ....?

Reference Frame 2: The Frame Where the Spaceship Stood Still

Now at this point, a lot of the Explainers of the Twin Paradox stop and say, see, everything works out. But stopping here is what causes the problems.

That's because the Doubters say, hold on. You only stuck with the stationary Earth-Planet reference frame. You have to prove that Relativity works if the Spaceship stays stationary and the Earth moves off.

And as we mentioned there are various ways to wave your hands and dismiss the topic. You can also use some fancy pants Space-Time diagrams and such stuff. But you rarely get a simple middle school algebra explanation like we saw for the Earth reference frame. This neglect is what has led even some famous scientists to say the two reference frames are irreconcilable and the theory is wrong.

For instance, not only do the Doubters point out the Twin Paradox requires the Twins to be both old and young at the same time, they reason thusly. If the Earth-Planet system is moving at 0.6c, then by the Lorentz factor being 0.8, the 6 light-year distance contracts to 4.8 light-years.

But for the Earth Twin to age 20 years overall, he must travel 6 light years on each leg of the trip, not 4.8!

Explain ***that,*** the Doubters say.

Well, it turns out we can. But we need to use another finding of Relativity.

That's the ***Relativity of Simultaneity***.

The More Things are Simultaneous ...

At this point you might go into spittle-flinging diatribes.

Now we're introducing ***another odd-ball finding of Relativity?***

What is this? Just another way to make the numbers balance out?

Well, not quite. The Relativity of Simultaneity is another inevitable conclusion of the speed of light being the same regardless of reference frame and middle school algebra. Then we'll find out that if two things happen at the same time in one reference frame, ***they do not happen at the same time in another reference frame***.

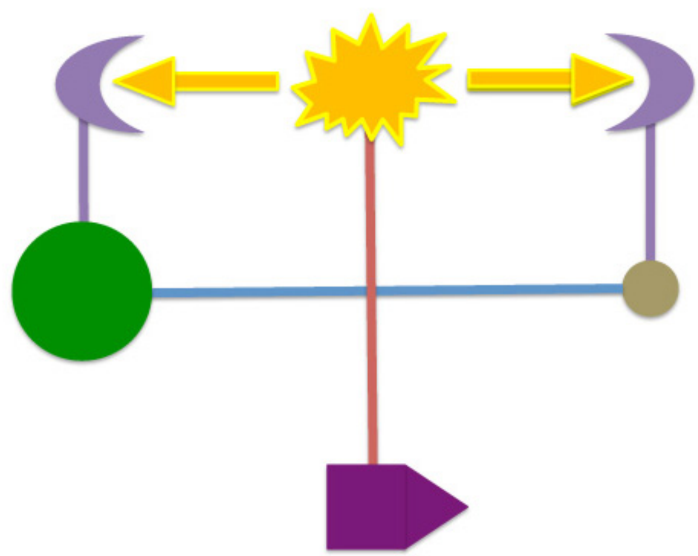
Now in the simplest introductions of Relativity, you almost never read about the Relativity of Simultaneity. But there's really no reason to omit what is as important as the other unusual findings of the theory. True, there are a few more steps in the math, making the bookkeeping a bit more involved. But it's all easy enough as we will see.

Consider the following. We'll put a beacon on the Spaceship. The beacon can at any time emit a flash of light. And on the Earth and the Planet, there are detectors to determine when the light reaches them.

Remember, the speed ***of the same beam of light*** will be measured ***traveling at the same velocity*** in both reference frames.

Now we ask, when do the Space Twin and the Earth Twin see the light reach the detectors?

It would seem that if you flash the beacon at the halfway point, the beams of light should reach the Planet and the Earth at the same time.



Simultaneous Events?

Is this what happens? Well, yes and no.

First let's look at the Spaceship as seen from the Earth-Planet reference frame.

The Earth and Planet are standing still. The Spaceship takes off, moves to the halfway point, and then it flashes the beacon.

So the beacon flashes when it is halfway between the Earth and Planet - 3 light-years from Earth. So by Earth's time the Spaceship has been traveling:

$$^{\textbf{3 light-years}}/_{\textbf{0.6 light-years per year}} \textbf{ = 5 years}$$

We emphasize, the speed of light is a ***constant*** regardless of the reference frame. Light does not pick up the speed of the source of the light. And your speed does not change the apparent speed of the light. Repeat. The speed of light ***as measured*** is a ***constant***.

So the beacon flashes and the two beams of light start off from the half-way point. One of the beams heads to Earth and the other to the Planet.

Both light beams travel (of course) at the speed of light for 3 light years. So the beams take 3 years to reach the Planet and the Earth.

From the Earth reference frame, then, the light hits the detectors at the same time. Counting from the beginning of the trip, that's in:

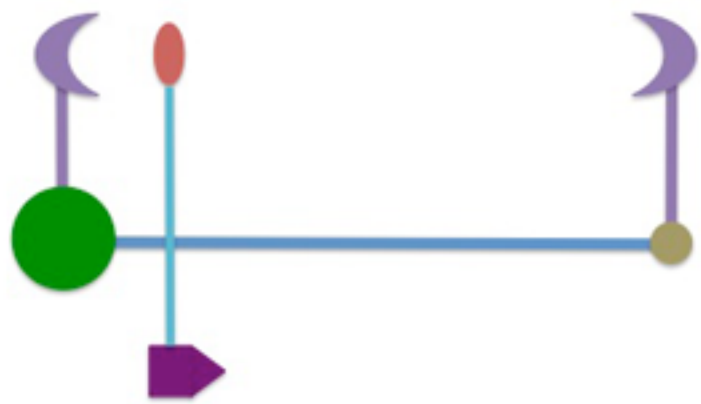
5 light years + 3 light years = 8 light years

And the Spaceship has to travel 2 more years to reach the Planet.

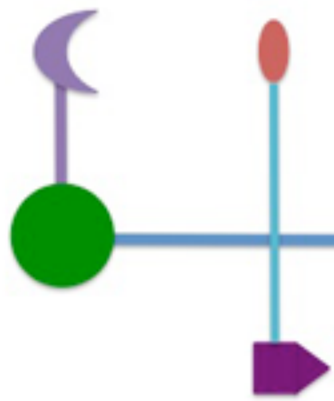
The following series of diagrams may make this a bit clearer.

(Note: In the web version of this essay, these diagrams are shown as an animation.

The Spaceship is on Earth.



And heads off.



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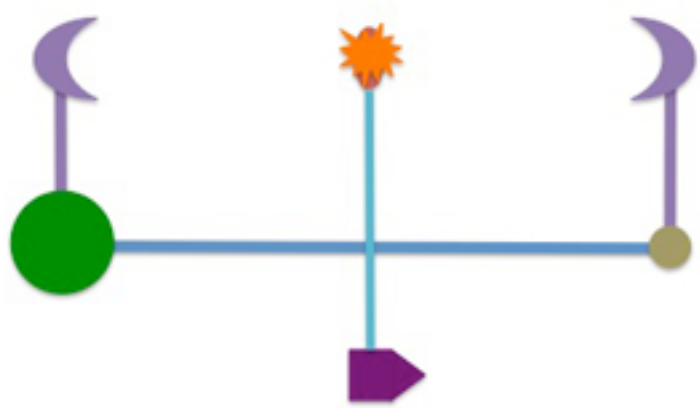
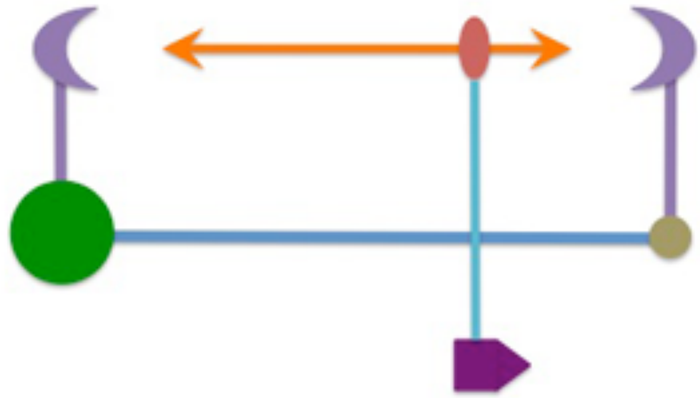
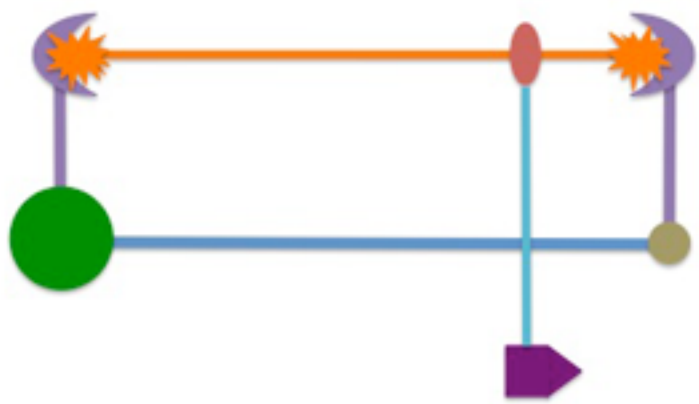


And the beacon flashes at halfway.



And continuing ...



And the Light Hits Both Simultaneously

And the Spaceship keeps going.

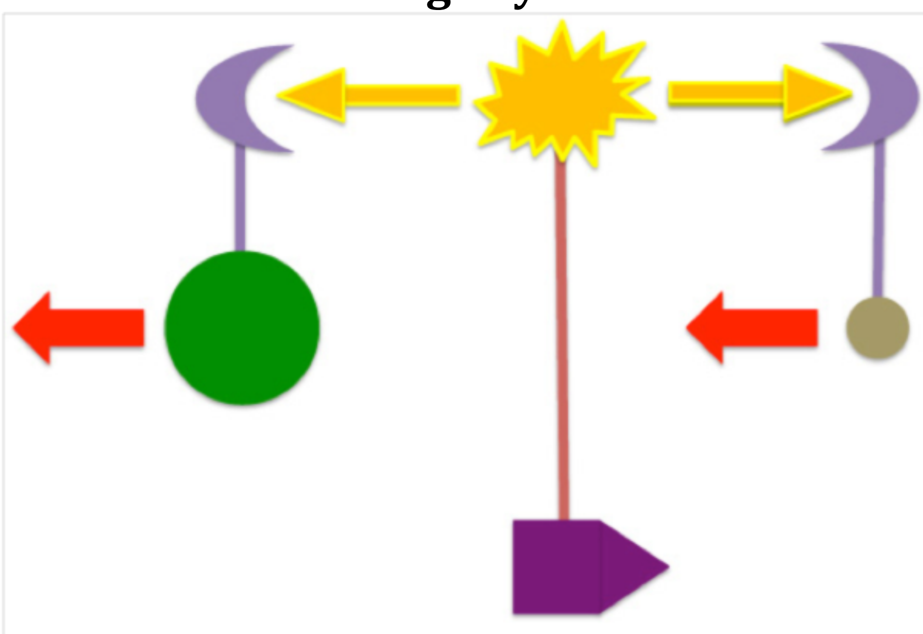
And finally reaches the Earth.

And from the Spaceship reference frame?

Well, from the Spaceship reference frame it's the Spaceship that is standing still. And the Earth-Planet "system" is moving. Since the Earth and Planet are moving, the distance between them contracts from the original 6 light-years to 4.8 light-years due to the Lorentz factor being 0.8. So the halfway point is 2.4 light-years.

Halfway Point for Space Twin = 6 light-years × 0.8 (Lorentz Factor) / 2

= 2.4 light-years.



Stationary Spaceship / Moving Planets

Now here's where things get interesting.

Remember from the reference frame of the Spaceship, the planet is moving ***toward*** the rocket. And so the beam of light is not only moving toward the planet, but the planet is also ***moving toward the approaching beam of light***. So the light beam will reach the planet quicker than if the planet was not moving.

How much quicker is pretty easy to calculate. Just divide the distance - 2.4 light-years - by the ***sum*** of the two velocities.

That is, the time needed for the light to reach the Planet ***after*** the light flash is:

$$\textbf{Time} = {}^{\textbf{2.4 light-years}}/_{\textbf{(1+0.6) light-years per year}}$$

$$= {}^{2.4}/_{1.6}\ \textbf{light-years}$$

$$= \textbf{1.5 years}$$

And since the Planet has been traveling for 2.4 light-years before the beacon flashed, it was already traveling for:

$${}^{\text{2.4 light-years}}/_{\textbf{0.6 light-years per year}} = \textbf{4 light-years}$$

And the total time it takes the light to reach the Planet from the start of the trip is:

$$\textbf{1.5 years + 4 years = 5.5 years.}$$

Not the same as the 8 years in the Earth reference frame.

Now things get ***veeeerrrrrrrryyyy*** interesting.

Remember that the ***Earth*** is also speeding ***away*** from the light source when the beacon flashed. So the light beam has to ***catch up with the Earth***. So the time will be ***longer*** than if the Earth was standing still.

To find the time, then, we divide the distance by the *difference* of the speed of light and the speed of the Earth.

So after the beacon flashes, the Earth travels:

$$\textbf{Time} = {}^{2.4\ \text{light-years}}/_{(1-0.6)\ \text{light-years per year}}$$

$$= {}^{2.4}/_{0.4}\ \textbf{years}$$

$$= \textbf{6 light-years}$$

And since the Earth has been moving for 4 years before the flash, the total time it takes the light to reach the Earth is:

4 years + 6 = 10 years.

So let's sum up what we've found.

Earth-Planet Reference Frame
(Spaceship is moving)

Time Light Reaches Earth: 8 years
Time Light Reaches Planet: 8 years

Spaceship Reference Frame
(Earth and Planet are moving)

Time Light Reaches Planet: 5.5 years
Time Light Reaches Earth: 10 years

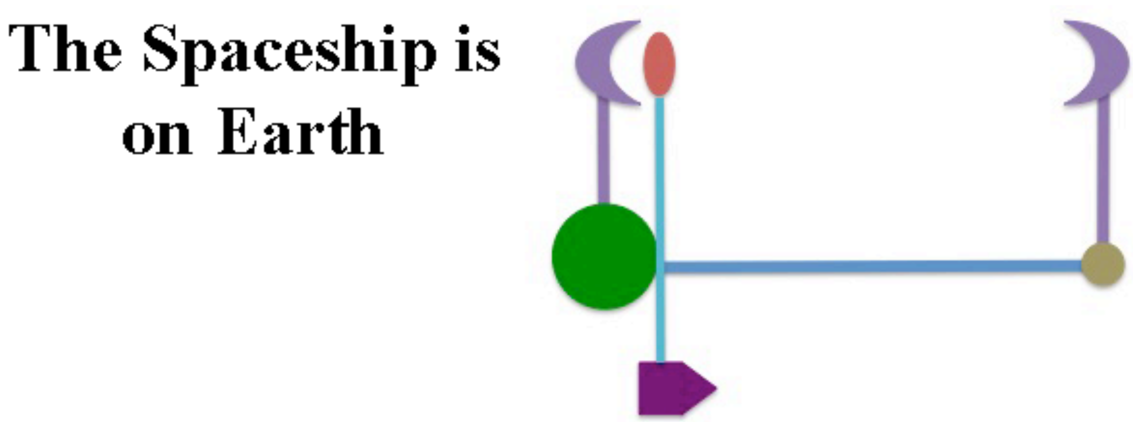
Ha? (To quote Shakespeare)

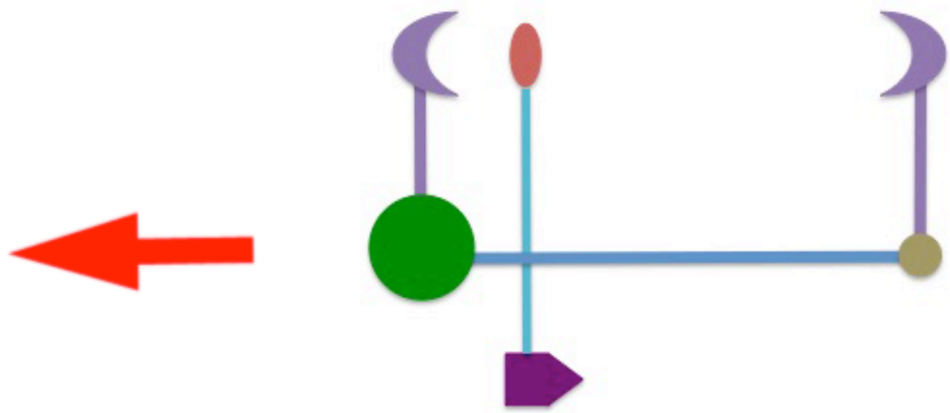
So it's true!

In one reference frame the light beams hit the Planet and Earth at the same time.

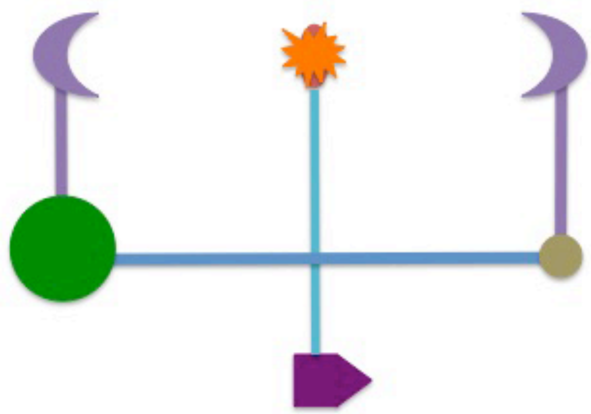
And in ***another*** reference frame, they ***don't!***

Yep. And to see ***this*** strange conclusion see the following diagrams.

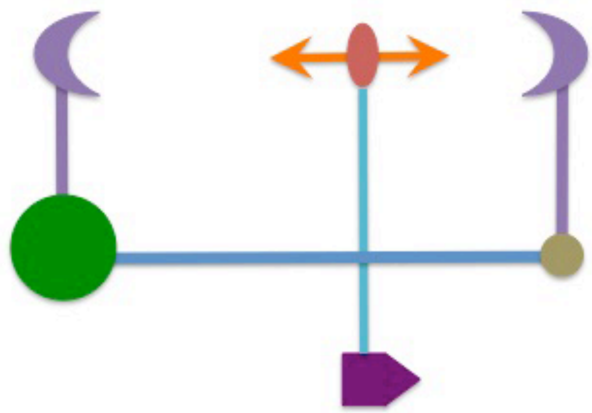




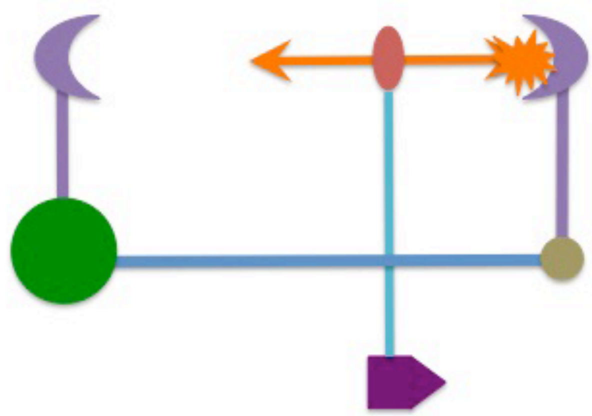
And the Earth (and Planet) head off.



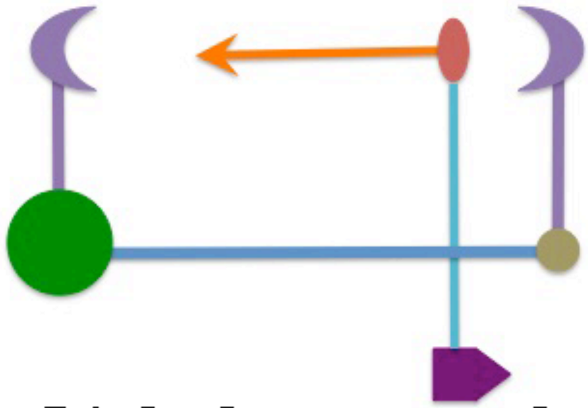
At the halfway point, the beacon flashes.



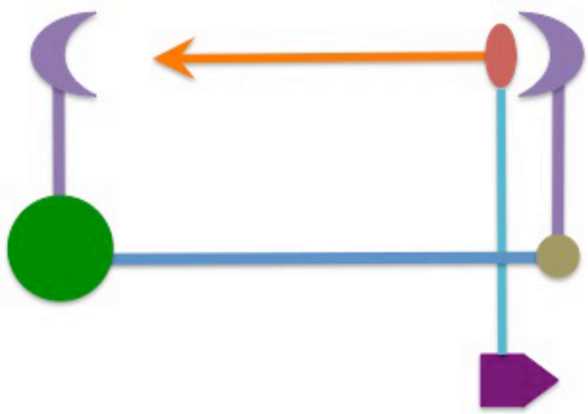
The Light Beams head out.



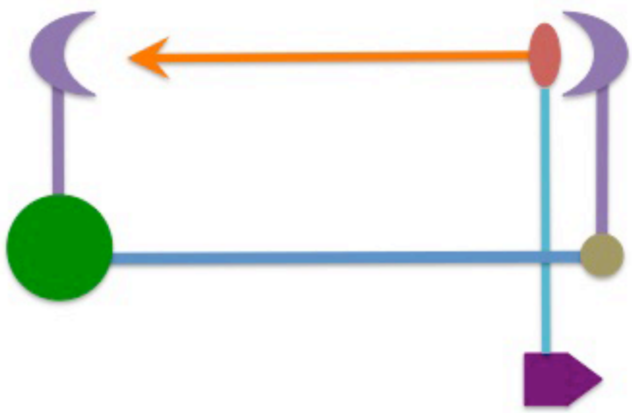
The Light Beam reaches the Planet.



And the Light has to catch up with the Earth



And continues ...



And keeps going ...

And finally the Light reaches the Earth

But if you watched carefully you might have noticed that as we calculated, the light reached the Earth in 10 years.

But the trip of the Planet to the Spaceship only took 5.5 years! And so if we use the Spaceship reference frame, the Earth kept going after the trip was over for the Planet!

But there's even something more unbelievable. You may have noticed that the Earth traveled 10 years - but the Planet only traveled for 8 years. But as hard as it is to believe, this isn't a problem.

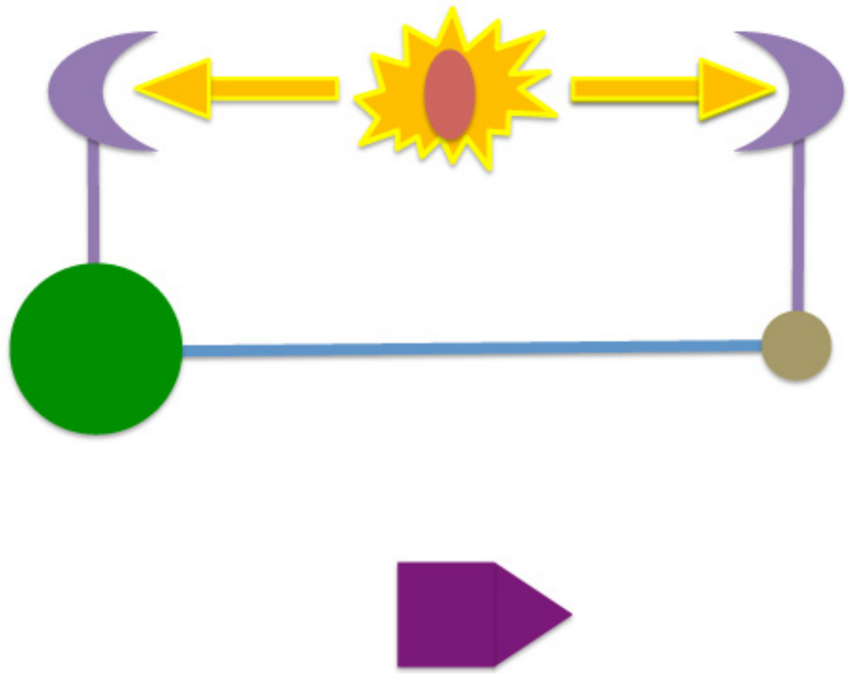
In fact, this ***Relativity of Simultaneity*** will make things work out.

Finally.

Making Relativity Work

We do, though, have to do things a little different.

First of all we will suspend the beacon halfway between the Earth and the Planet. And it can be set off by either the Space Twin or the Earth Twin by sending a radio signal. Of course, the signal travels at the speed of light.



The New Arrangements: Beacons in Space!

As a simplification we'll only consider the outward leg of the trip realizing the return trip takes the same length of time.

What we will do is have the Earth delay sending the radio signal long enough so that when the beacon flashes ***the light beam will reach the Planet at the same time as the Spaceship reaches the planet.***

First we'll use the Earth-Planet reference frame. That is, the Earth and the Planet are motionless, and the Spaceship is in motion.

Now what we do is to have the Earth Twin wait 4 years after the Space Twin takes off before he sends the signal. And remember the beacon sits halfway between the Earth and the Planet distance of 6 light-years. So the radio signal travels half the distance - 3 light-years - and sets off the flash.

So the light beam reaches the beacon and the beacon flashes:

$$\textbf{Time = 4 years + 3 years}$$

$$\textbf{= 7 years}$$

after the start of the trip.

And the light beam from the beacon must travel ***another*** 3 light-years to reach the Planet - or 3 more years. So after the Spaceship takes off, the light beam reaches the planet in a total of:

$$\textbf{Time = 7 years + 3 years}$$

$$\textbf{= 10 years}$$

But remember. The Spaceship travels at 0.6c for 6 light-years. So it also reaches the Planet in:

$$^{\textbf{6 light years}} / _{\textbf{0.6 light-years per year}} \textbf{ = 10 years}$$

So we are correct in our calculations. From the Earth-Planet reference frame, ***both*** the light beam and the Spaceship reach the Planet at the same time, after 10 years.

And when does the light reach the Earth?

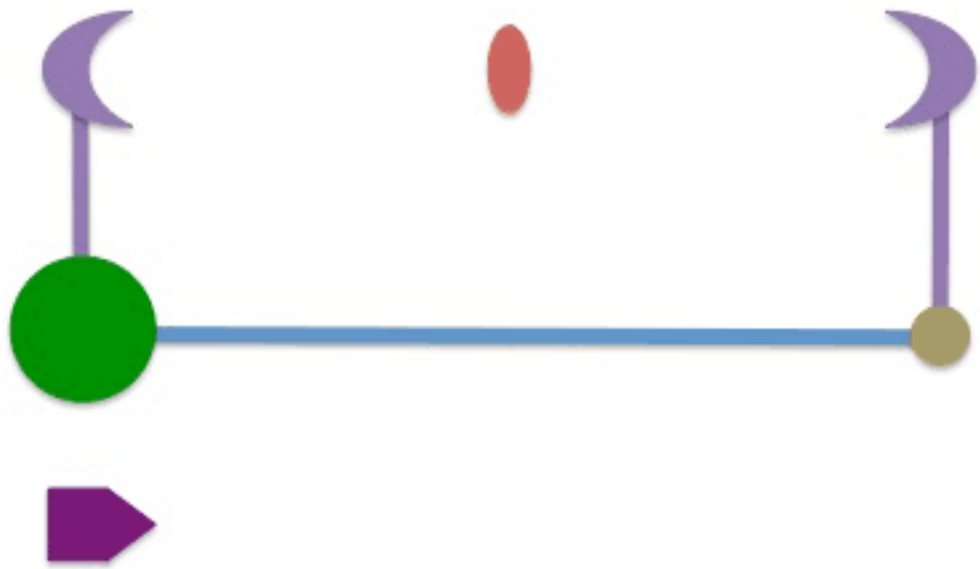
Again the trip has been going on for 7 years before the beacon was flashed. And the light again travels for 3 more years.

So the light reaches the Earth in:

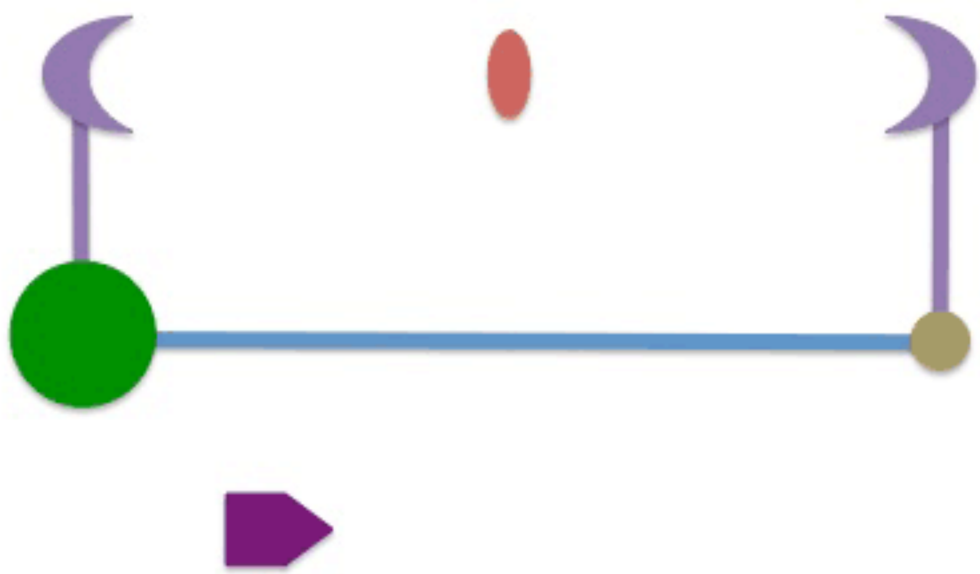
Time = 7 years + 3 years

= 10 years

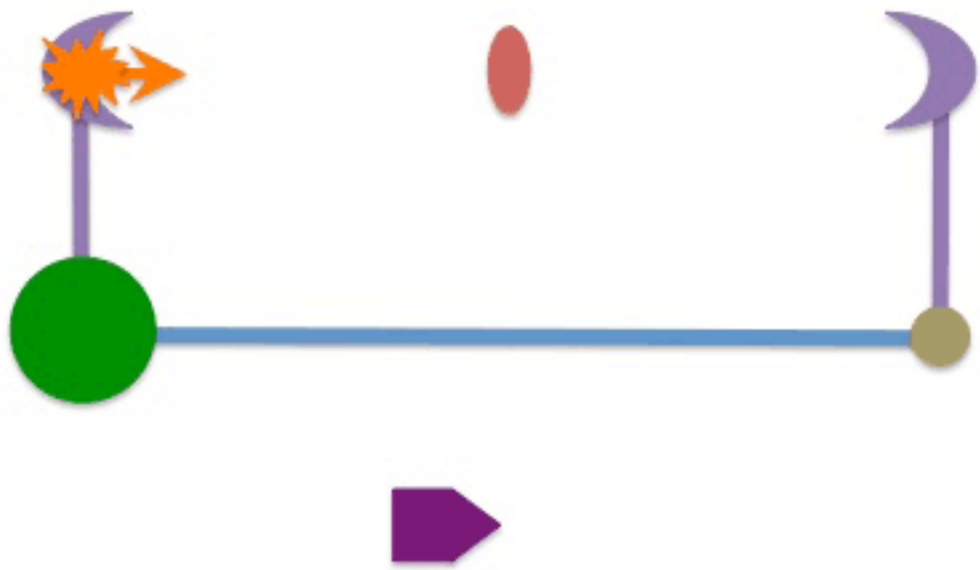
As before we'll use some diagrams to help ease comprehension.



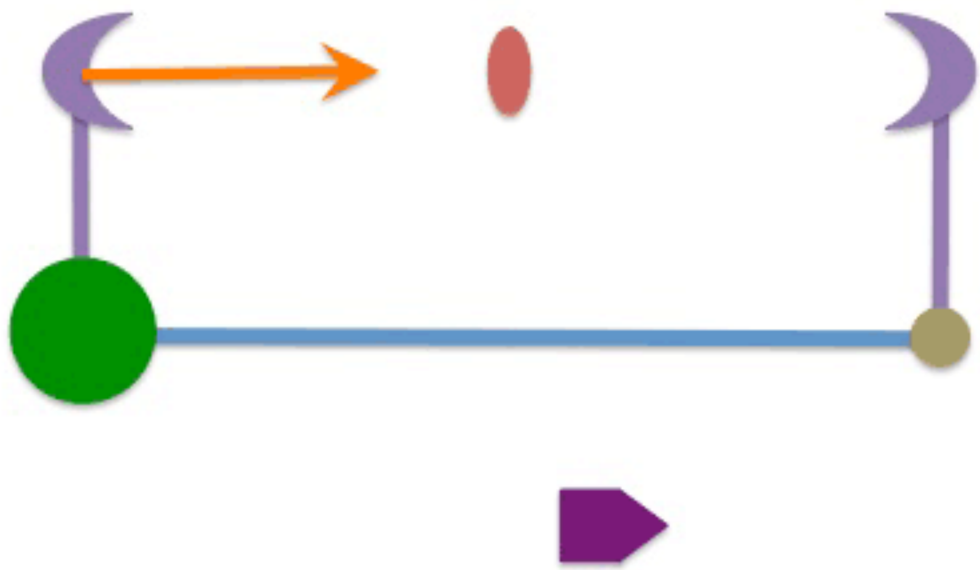
The trip begins!



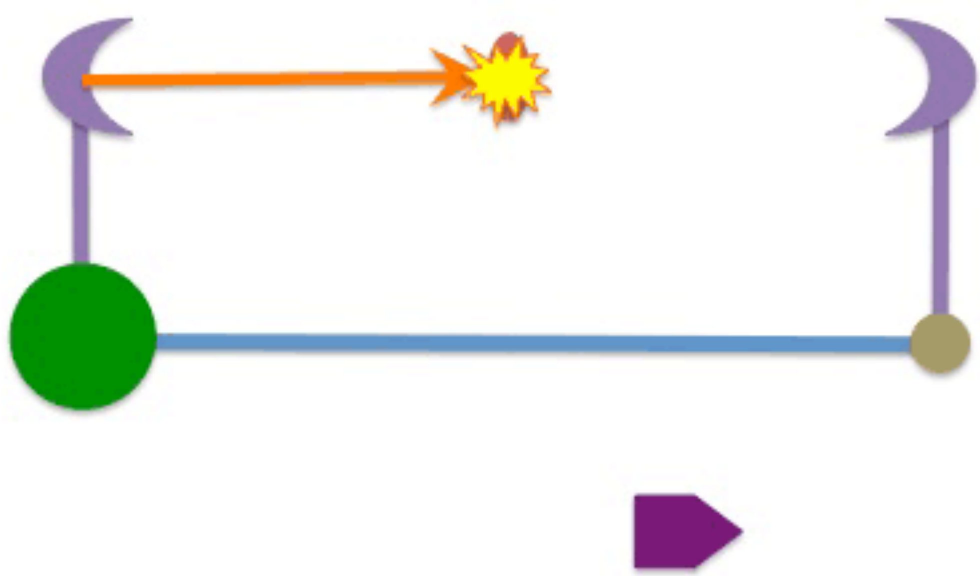
And the Spaceship continues!



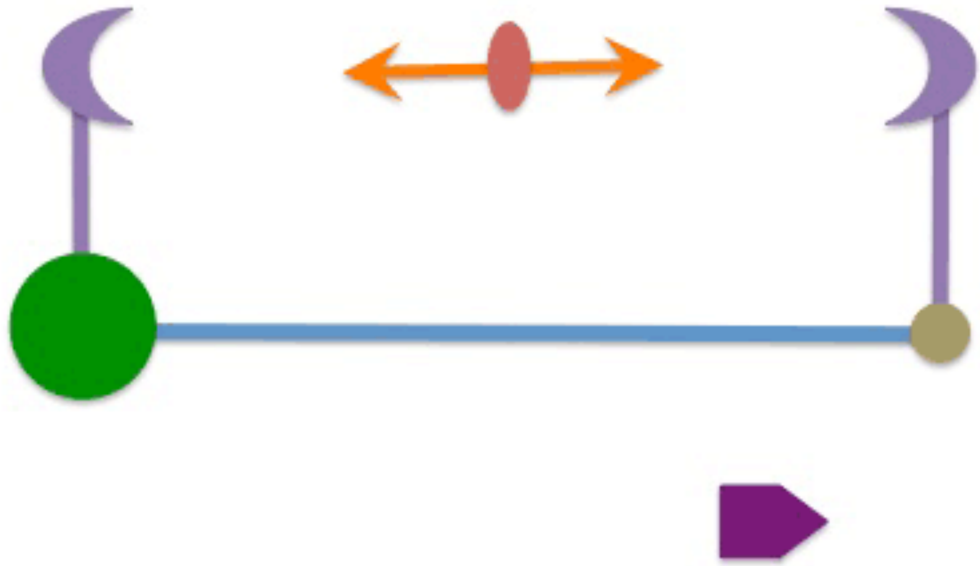
After 4 Years, the Space Twin Sends a signal to the Beacon



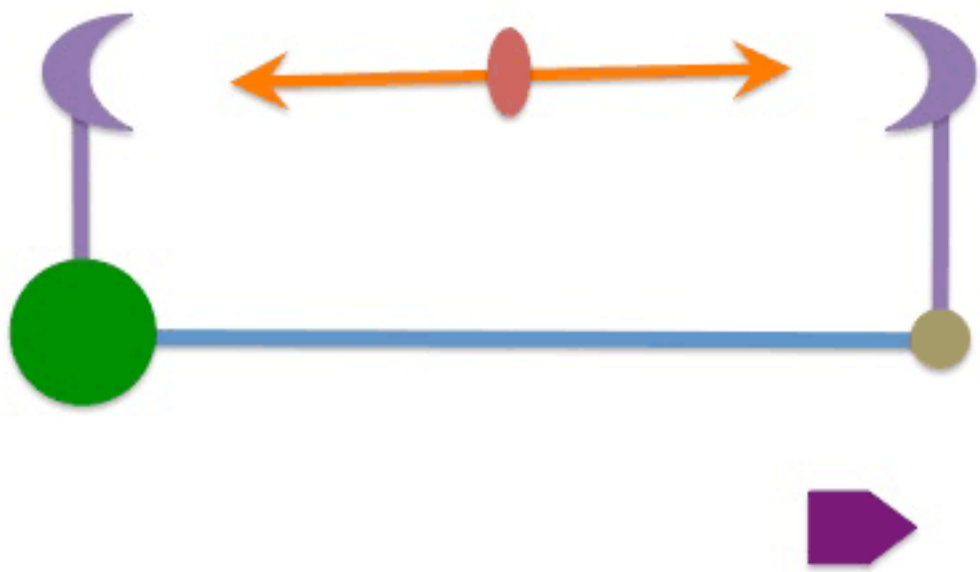
And the signal approaches the Beacon while the Spaceship continues toward the Planet.

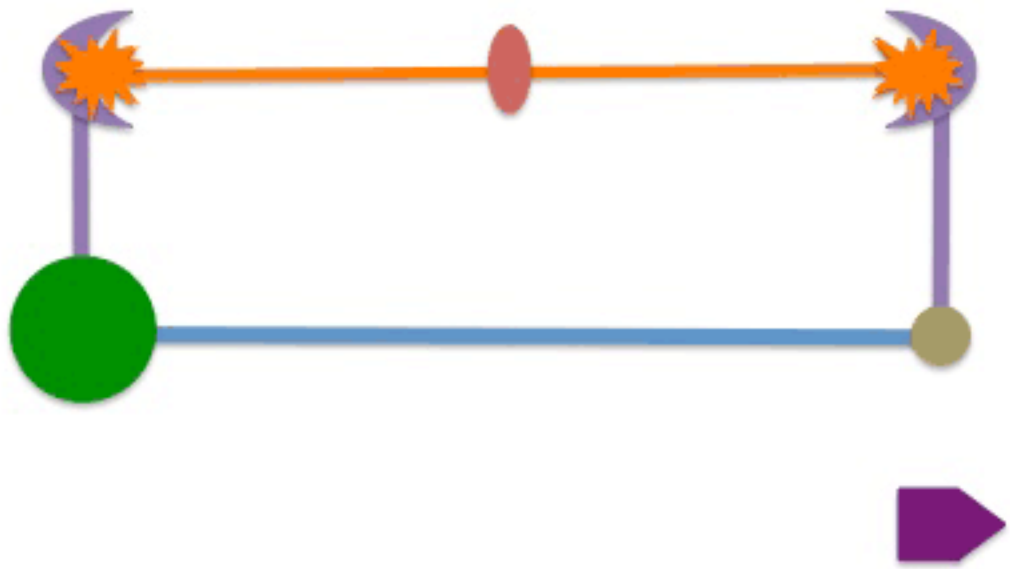


And the signal reaches the beacon after 3 years.



And the light beams head out.





And the light beams reach the Earth and the Planet in 3 more years and at the same time when the Spaceship reaches the Planet.
Time for First Leg: 10 years

Return Trip: 10 Years
Total Time: 20 Years

So the two beams of light reach the Earth and the Planet at the same time and after 10 years. This agrees with our earlier calculation using the stationary Earth-Planet system. Of course, the return trip would also take 10 years for a round trip of 20 years.

To summarize so far, for a stationary Earth-Planet system and a moving Spaceship

The Earth Twin sends the signal to the beacon after 4 years. The radio signal takes 3 years to reach the beacon and so the beacon flashes after 7 years.

The light reaches the Earth and Planet at the same time, after 10 years. This is also the time the Spaceship reaches the Planet.

The return trip takes the same length of time as the first leg of the trip.

Which brings us to the conclusion:

The Earth Twin sees the Spaceship return to Earth after a total of 20 years by the Earth clock.

Of course, if the Earth Twin were to monitor the Spaceship's clock, Space Twin would experience only 8 years for the first leg of the trip or 16 years for the round trip.. So we also conclude:

The Earth Twin determines the Space Twin is 4 years younger after the end of the trip.

OK. Now let's switch to the Spaceship reference frame. That is the Earth and the Planet start moving while the Spaceship stays stationary.

First the (moving) Earth starts the trip by moving away from the (stationary) Spaceship. The Earth Twin still sends his signal four years after the trip begins - but of course based on the Earth clock. The signal to flash the beacon travels at the speed of light.

Now since the ***Earth Twin*** is moving, his time appears slower when seen by the stationary Space Twin. And remember that the Earth Twin sent the signal after 4 years of his time.

Now remember the time dilation equation:

$$t_2 = t_1\sqrt{1 - (^v/_c)^2}$$

In this case, we know the time for the ***moving*** Twin, but want to calculate the time the ***stationary*** Twin sees. So we must ***divide*** the time of the Earth Twin by the Lorentz factor.

$$t_1 = {}^{t_2}/_{\sqrt{1 - (^v/_c)^2}}$$

And this gives us:

Time for Space Twin = 4 years (Earth Twin's Wait) / 0.8 (Lorentz Factor)

= 5 years

So for the Space Twin, the Earth Twin waited 5 years - not 4 - before sending the signal to the beacon. Once sent, the signal travels toward the beacon at the speed of light, **1c**.

Also remember that in the Space Twin's reference frame, the distance between the Earth and Twin is not 6.0 light years. And because the Earth and Planet are moving, the distance must be ***multiplied*** by the Lorentz factor.

Time for Space Twin = 6 light-years (Earth Twin's Distance) × 0.8 (Lorentz Factor)

= 4.8 light-years

Therefore since the beacon is halfway from the Earth to the Planet, the beacon is

$$\textbf{Earth-Beacon Distance (Spaceship Reference)} = {}^{\text{4.8 light-years}} / {}_{2}$$

$$= \textbf{2.4 light/years}$$

from the Earth and from the Planet.

Now here we have to be ***very*** careful with out bookkeeping. We are encountering a situation like we did when considering the Relativity of Simultaneity and have similar calculations to contend with.

Since the beacon is ***moving toward the approaching radio signal*** at 0.6 c, and the radio signal is approaching the beacon at 1c, the radio signal reaches the beacon in:

$${}^{\text{2.4 light-years}} / {}_{\text{(1 + 0.6) light-years per year}}$$

$$= \textbf{1.5 years}$$

after the signal is sent from the Earth.

Remember, for the Spaceship the time the Earth Twin delayed sending the signal was 5 years, not 4. And the time it takes the radio signal to reach the beacon is 1.5 years.

So the Space Twin sees the beacon flash in:

$$\textbf{5 years + 1.5 years = 6.5 years}$$

after the start of the trip.

But once the beacon flashes, the light beam also travels 2.4 light years toward the Planet.

But again be careful! The Planet is also ***moving toward the light beam*** at 0.6c.

So for the Space Twin the time for the light to reach the Planet is also:

$$^{\text{2.4 light-years}} / _{\text{(1 + 0.6) light-years per year}}$$

$$= {}^{\text{2.4 light-years}} / _{\text{1.6 light-years per year}}$$

$$= \text{1.5 years}$$

So counting ***from the start of the trip***, the light reaches the Planet in

6.5 years (Time of Flash After Start of Trip) + 1.5 years = 8.0 years.

And as the Planet is moving toward the Spaceship at 0.6c and travels for 4.8 light-years. So the Planet also reaches the Spaceship in:

$$\text{4.8 light-years} \,/\, \text{0.6 light-years per year}$$

= 8.0 years

So again we are correct. If the beacon flashes after 5 years as measured by the stationary Space Twin, the Planet reaches the Spaceship at the same time the light reaches the Planet. So the first leg of the trip for by the Space Twin's clock is 8 years.

Exactly as we first saw with the stationary Earth-Planet reference frame.

OK. Here's where things change.

Just what does the ***Space Twin*** see is happening for the Earth Twin? Specifically, when does the Space Twin see the light reach the Earth and so mark the end of the first leg - the halfway point - of the whole trip ***for the Earth Twin?***

Well, the Earth is moving ***away*** from the light beam at 0.6 c. And the light is ***catching up*** at 1c. And since the Earth is 2.4 light-years away at the flash, it takes:

$$\textbf{6.5 years (Time of Flash)} + {}^{\textbf{2.4 light-years}} / {}_{\textbf{(1 - 0.6) light-years per year}}$$

$$= \textbf{6.5 years} + {}^{\textbf{2.4 light-years}}/{}_{\textbf{0.4 light-years per years}}$$

$$= \textbf{6.5 years + 6.0 years}$$

$$= \textbf{12.5 years.}$$

Ha????!!!!!?????? (again Shakespeare).

12.5 years????!!!!!??????

Yes. 12.5 years.

But we just calculated the first leg for the Space Twin was 8 years!

That is correct.

So you're saying the Planet ends its first half of the trip before the Earth does?

Exactly.

So things are more complicated than ever!

Weeeeellllllllll, not quite.

In fact, things are actually working out nicely.

First we shouldn't be too upset that the Earth and Planet end the first leg of their trips at different times in the Spaceship reference frame. We showed earlier that events which are simultaneous in one reference frame are not simultaneous in another. If the Earth and Planet end the first half of the trip simultaneously in the Earth-Planet reference frame, that need not be true in the Spaceship reference frame.

And besides this extra 4.5 years is just what we need!

For the Spaceship Twin, the Earth Twin doesn't end its first half of the trip for 12.5 years. But we also know that as measured on the Spaceship, time slows down on Earth since in our current reference frame the Earth and Planet are moving and the Spaceship is stationary.

And we know that the amount that time slows down on a moving Earth is calculated by the Lorentz factor. For an Earth traveling at 0.6c the Lorentz factor is 0.8. So the 12.5 year trip as seen by the Space Twin, will mean the Earth-Twin's clock reads:

Trip Duration (for Earth Twin) = 12.5 years (Rocket Reference Frame) × 0.8 (Lorentz factor)

= 10 years

What was that number again?

10 years!

You mean the same time it takes as measured by the Earth Twin if the Earth and Planet are standing still and the Spaceship is moving?

Yep, that's exactly what we mean.

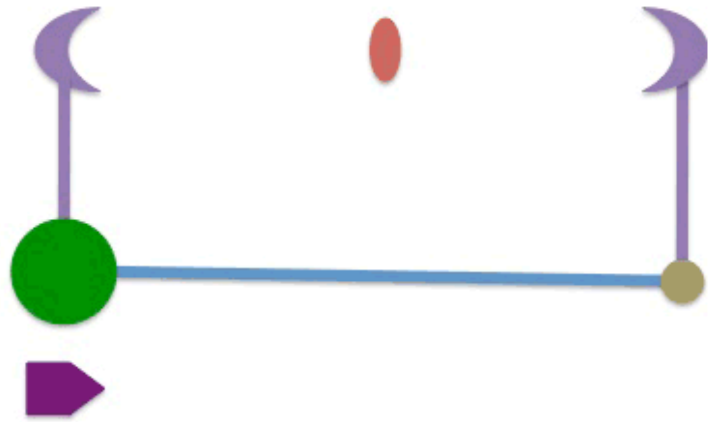
So with a stationary Spaceship reference frame, we see that everything is consistent with the stationary Earth-Planet reference. Time did indeed slow down on the Moving Earth-Planet system and distances contracted - ***and by the amount dictated by Special Relativity!***

But ***because of the Relativity of Simultaneity***, the Earth Twin keeps going another 4.5 years and so still experiences a 10 year first half of the trip.

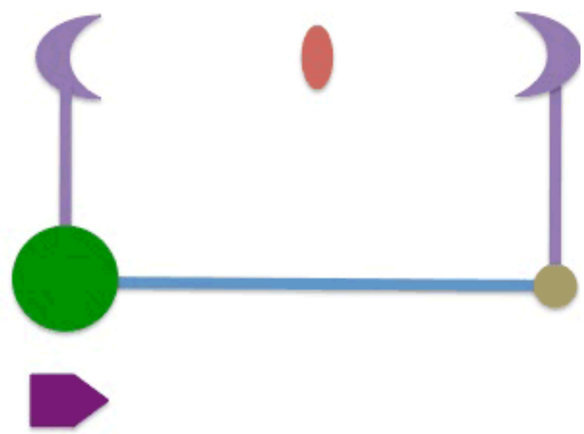
That means that both Twins agree that the Earth Twin's clock reads 10 years when the Earth Twin begins ***his*** return trip. And they agree that the Space Twin's clock reads 8 years!

Calculating the time for the return trip is more complex in the Spaceship reference frame because of the Relativity of Simultaneity. Because the first leg of the trip does not end at the same time for the Spaceship and Planet, we deviate from the constraints of uniform velocity in a straight line needed for the calculations. The best way to handle it would be simply to have everyone pause after the first leg of the trip for a certain interval. That way the Spaceship, Planet, and Earth "reset" to the same reference frame before beginning the return trip. The second trip would then be a mirror image of the first leg. So the elapsed time during the trip would then be 20 years for the Earth-Planet Reference frame and 16 for the Spaceship.

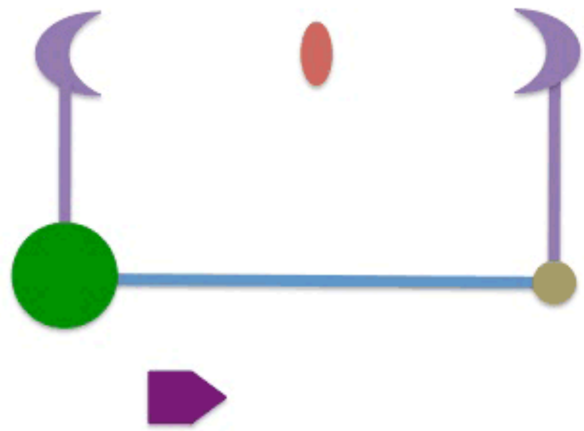
Once more we'll go through these steps diagrammatically.



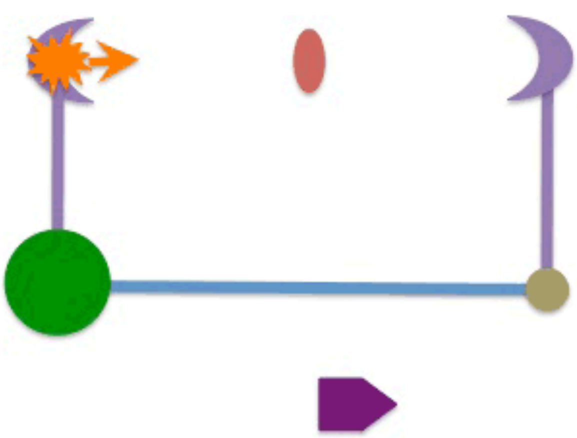
First the Earth, Planet, and Spaceship are ready to go. Distance between the Earth and the Planet is 6 light-years.



But once the trip begins, the distance contracts to 4.8 light-years due to the Lorenz Factor.

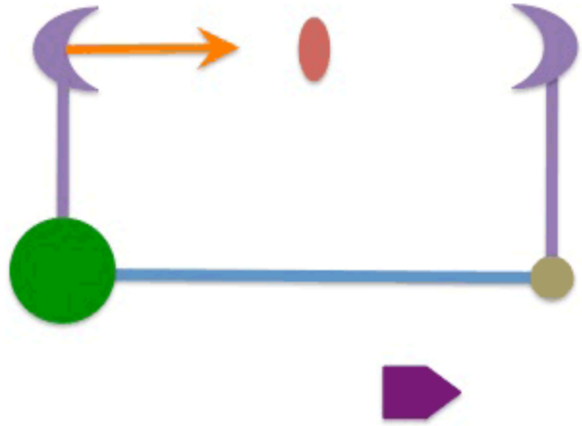


And the Earth and Planet take off.

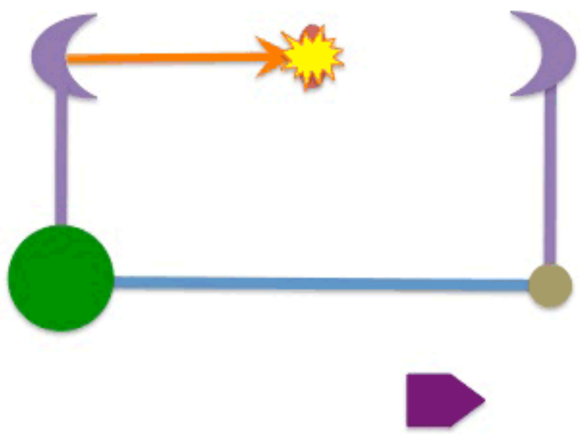


And the Earth Twin sends the signal - not after 4 years but after 5 years due to the Lorenz Factor

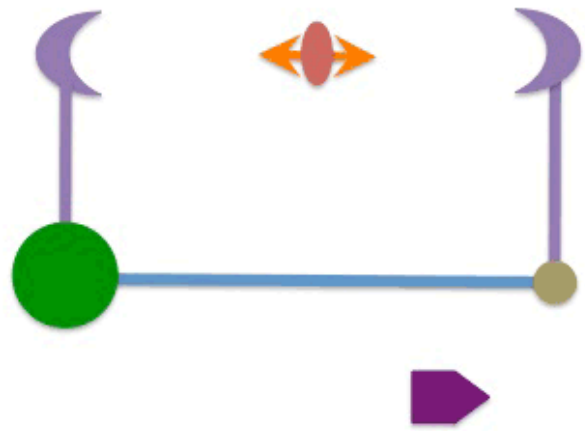
4 Years ÷ 0.8 (Lorentz Factor) = 5 Years



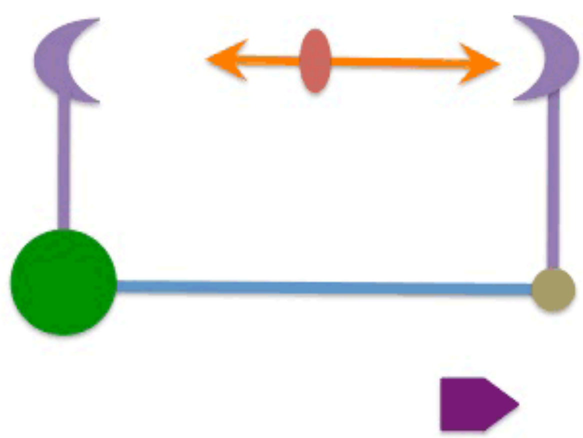
And the signal heads toward the Beacon.



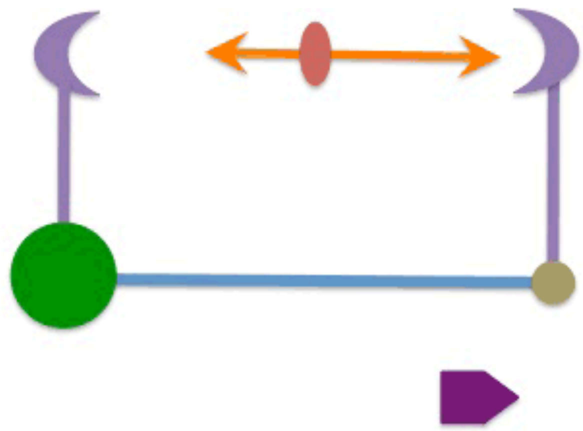
But since the Beacon is heading toward the light beam,
The two meet in 1.5 years - not 3.



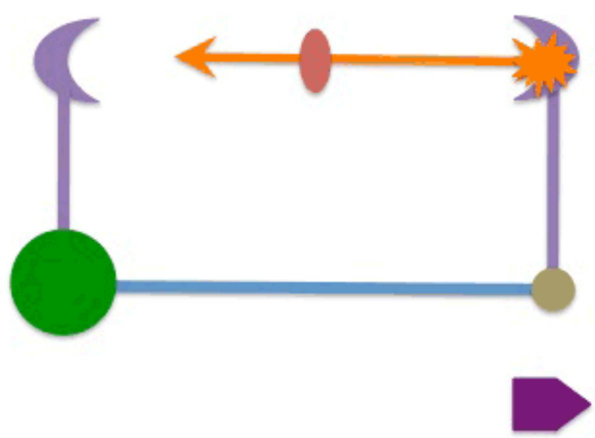
And the light beams head out to the Earth and the Planet.



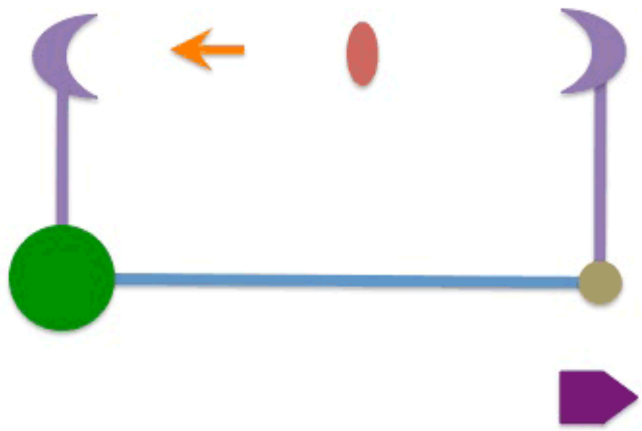
But because the Planet moves toward the light beam, it reaches it ...



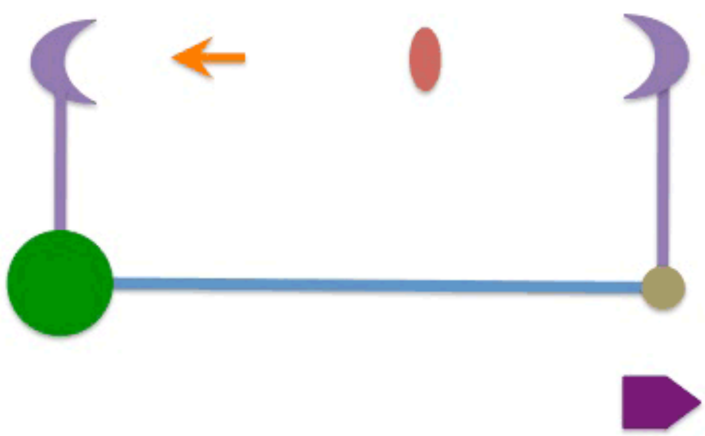
But because the Planet moves toward the light beam, it the two meet in ...



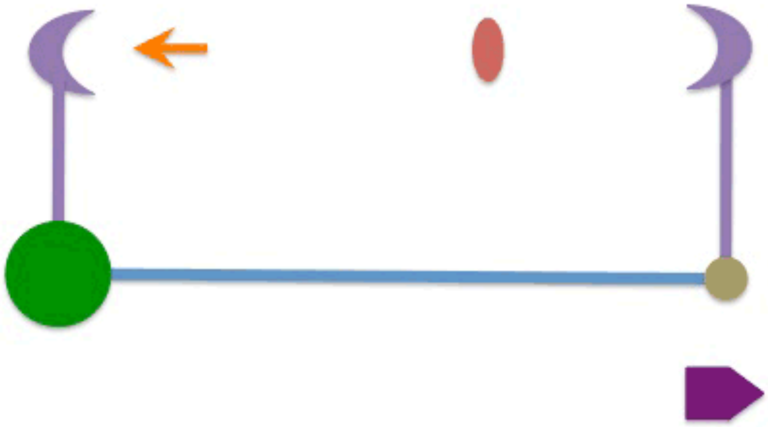
Another 1.5 years.
Total Time: 5 + 1.5 = 6.5 years
The same time the Planet
reaches the Spaceship and the
first leg of the trip ends for the Spaceship ...



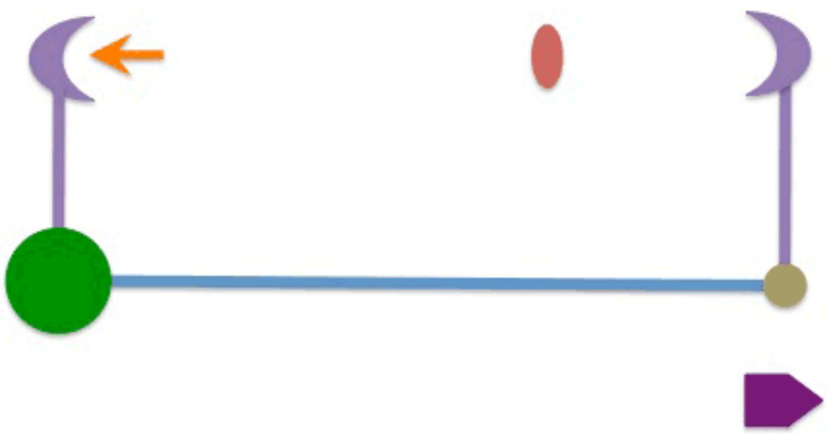
But not for the Earth!
Because the Earth was moving away from the beam of light, it keeps going ...



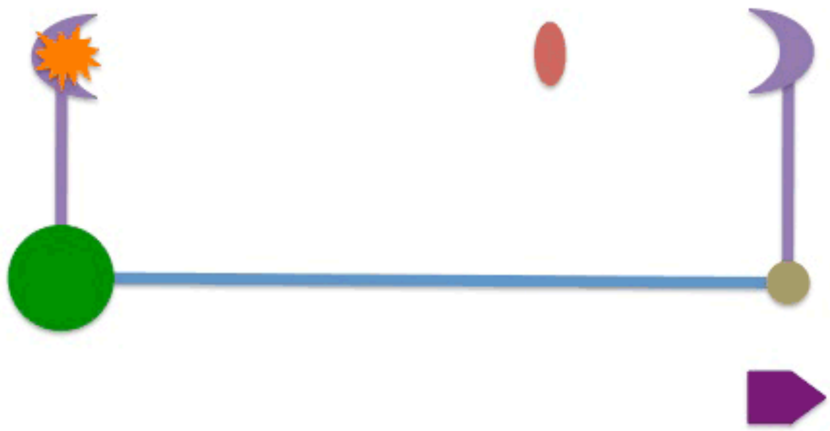
and going ...



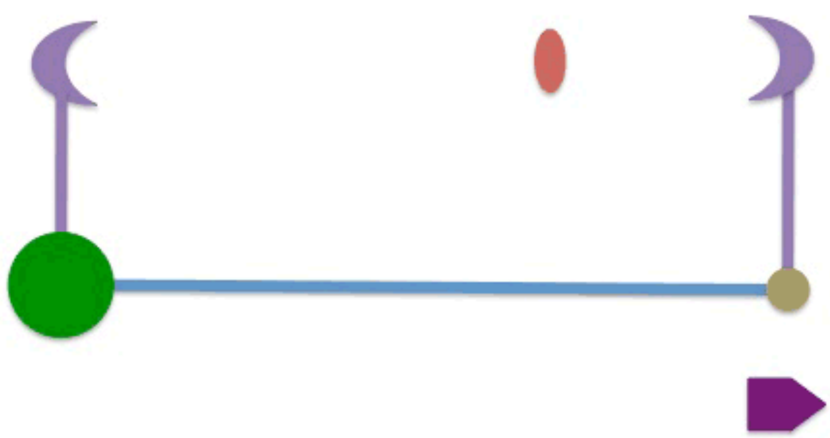
and going ...



and going ...



and reaches the Earth after 12.5 years - but according to the Spaceship's clock!



Which by the Earth clock is:
12.5 Years X 0.8 (Lorentz Factor) = 10 Years

Exactly the time calculated from the Earth reference frame!

And the return trip will double the time!

So in the reference frame where the Spaceship stays still and the Planet and Earth are moving, the Principles of Relativity still have the Earth Twin age 20 years and the Space Twin ages 16 years.

There's a message to remember as it is central to understanding Relativity. If light acted like baseballs, then light would pick up or lose the velocity of its reference frame. In that case there would be no difference in the times calculated from either a stationary Earth-Planet system or a stationary Spaceship reference. But the speed of light does not pick up or lose any extra velocity from its reference frame, and we know this because it is an experimental fact. The rest is all middle school algebra.

And some people may worry how to ***interpret*** what happens when the Planet ends its first leg and starts the return trip and yet the Earth keeps going. It's almost like the space between the Earth and Planet get stretched like a rubber band. But this concept should not be any more of a problem than the idea of the Lorentz contraction - it just goes the other way. Relativity does introduce that strange notion of distortion of space which becomes a key factor in General Relativity and the effect of gravity. But as everything is distorted in proportion, nothing changes within the reference frame itself.

In any case, regardless of which reference frame you choose, the Space Twin is 4 years younger than the Earth Twin at the end of the round trip.

How about that?

Et In Conclusione?

As we said at the beginning, if you accept 1) the measured constancy of the speed of light and 2) middle school algebra, then Relativity is an inescapable consequence.

In fact, the high quality of the experimental data and the simplicity of the mathematics is why Relativity was accepted so quickly. By 1919, virtually all physicists accepted what was then still called a "theory", and despite what you may read on - shall we say "certain websites" on the Fount of All Knowledge - they still do.

Every now and then you may hear that someone ran an experiment and found that the difference observed in the speed of light does depend on where you shine the beam. These experiments have inevitably been shown to be non-reproducible by others or that the results don't disprove Relativity after all. And sometimes it's simply that the experimental error has been underestimated, something that's easy to do.

In 1915, Albert published another paper. This was about the ***General Theory of Relativity***. In this case Albert pointed out there was no difference in the force you feel if you accelerate and the force you feel due to gravity. Then using some more math - a bit more advanced than middle school - Albert made some more predictions. One was that we should see light bend if it passes by a massive object.

In 1919, astronomers took some pictures of the stars that show up near the Sun during a total eclipse. The stars weren't exactly where they should have been. But the apparent shift in position was what Albert's theory predicted if light was being bent.

It was also known that the planet Mercury didn't quite behave like Isaac Newton said it should. When it completed an orbit it didn't ***quite*** return to where it started. Sure enough, General Relativity predicted the difference and that it was what Albert said it should be. These experiments made headlines and suddenly Albert and the Theory of Relativity was famous.

People kept finding support for Albert's ideas. In 1964, two scientists took a cesium clock - the most accurate kind we have - and put it in a plane. They flew around the Earth and compared the time to a clock that stayed on the ground. Although the changes were in nanoseconds, the differences were there and agreed with what both the Special and General Theories of Relativity.

And more recently the world has found not just ***support*** for Albert's theory, but there's actually a practical use for it. For a Global Positioning Satellite to remain at one spot above the surface of the Earth, it actually has to move 3285 miles per hour faster than the rotation of the Earth's surface. And because the satellites are 12,000 miles above the Earth, they experience a different pull of gravity and have to be corrected for both the Special and General Theory of Relativity. So without Albert, the annoying voice would be telling you to "Turn left! Turn left! Turn left!" about five miles from where you're supposed to. And it would get worse each day.

But suppose - just suppose - that someone eventually ***does*** show there really ***is*** a difference in the speed of light depending on reference frame? Well, if they ever do, then the difference will be so small - tiny, even eentsy - that the basic conclusions of Relativity will still hold. Time will still slow down for a faster moving clock, and length will still get shorter, and simultaneity will still be relative - but just be a smidgen less than Hendrik's equations predict.

In fact, Relativity has hit the point of many correct theories. Eventually they become fact from an epistemological standpoint. So that's why we can legitimately say the ***Principle*** of Relativity rather than the ***Theory*** of Relativity. After all it's kind of ridiculous to say that Relativity isn't true when the world - including our cars - acts like it is.

But if Relativity ***still*** seems a bit too complex and strange - or if all this middle school algebra is a bit too much - as an American president once said:

"Trust me!"

References and Further Reading

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"Einstein Explains Relativity", *Life Magazine*, April 11, 1938, p. 49.

"Cardinal Sees Atheism's Ghost Behind Relativity", the Miami *Herald*, April 9, 1929. A complaint that Albert's theory may be promoting atheism - from an American.

The Modern Intellectual Tradition: From Descartes to Derrida, Professor Lawrence Cahoone, Ph.D., Professor of Philosophy, College of the Holy Cross, The Teaching Company. Gives a definition of ***Relativism*** which has nothing to do with ***Relativity***.

The Master of Light: A Biography of Albert A. Michelson, Dorothy Michelson Livingston, University of Chicago Press, Chicago (1979). Albert found light didn't change speed depending on the reference frame.

"Ole Roemer and the Speed of Light", *American Museum of Natural History*, http://www.amnh.org/education/resources/rfl/web/essaybooks/cosmic/p_roemer.html. The first measurement of the speed of light.

"Einstein at the Patent Office", *Swiss Federal Institute of Intellectual Property*, https://www.ige.ch/en/about-us/institute.html. Nice bio of Albert's early years.

How Does Relativity Theory Resolve the Twin Paradox?, Ronald Lasky, *Scientific American*, March 17, 2003. This article used the 0.6c velocity and the 6 light-years distance since as we saw this simplifies the numbers. It also shows you - again in an interesting manner - what the Twins would actually see in real time.

However, as one commenter posted, there was no explanation of the actual question always asked - how to reconcile the Twin Paradox if the Spaceship is treated as stationary. We have seen in our own essay that you must introduce the Relativity of Simultaneity - and, alas, the extra explanation lengthens the size of the essay beyond what is desired for modern magazine articles.

Viva la Internet and online publishing!

Relativity: An Introduction to Spacetime Physics, Steve Adams, Taylor and Francis, 1997. A comprehensive introduction of both the Special and General Theory.

"Scientist and Mob Idol", Alva Johnston, The *New Yorker*, December 2 - 7, 1933. A rather rambling and not terribly informative "profile" of Albert Einstein. But we see that claims of Albert being famous only after World War II are not correct.